SENIOR HIGH MATHEMATICS

MATH 14

COURSE OF STUDIES

IMPLEMENTATION - SEPTEMBER 1989

CURRICULUM DESIGN BRANCH

Alberta
EDUCATION

A. PROGRAM RATIONALE AND PHILOSOPHY

In order for students to be able to set goals and make informed choices, they need an array of thinking and problem-solving skills. Fundamental to this is an understanding of mathematical techniques and processes that will enable them to apply the basic skills necessary to address everyday mathematical situations, as well as acquire higher order skills in logical analysis and methods for making valid inferences.

A knowledge of mathematics is essential for a well-educated citizenry. However, the need for and use of mathematics in the life of the average citizen is changing. Emphasis has shifted from the memorization of mathematical formulae and algorithms toward a more dynamic view of mathematics as a precise language, used to reason, interpret and explore. There continues to be a need for the logical development of concepts and skills as a basis for the appropriate use of mathematical information to solve problems. Moreover, the use of available technology along with techniques such as estimation and simulation, incorporated with more traditional problem-solving techniques, are the tools by which mathematical problems are solved.

Change in the way in which mathematics is used is necessitating a concurrent change in the emphases of mathematics education. Students need an expanded list of fundamental concepts but will also need to understand the ideas that make up those concepts and how they are related. They also require a familiarity with their applications. Most important, students have to be able to solve problems using the mathematical processes developed, and be confident in their ability to apply known mathematical skills and concepts in the acquisition of new mathematical knowledge. In addition, the ability of technology to provide quick and accurate computation and manipulation, to enhance conceptual understanding and to facilitate higher order thinking, should be recognized and used by students.

The majority of students who enter high school exhibit mainly concrete operational behaviours with regard to mathematics. It is recognized that high school mathematics courses include many abstract understandings which students are expected to acquire. The course content of the high school mathematics program is cognitively appropriate for the students and should be presented in a way which is consistent with the students' ability to understand.

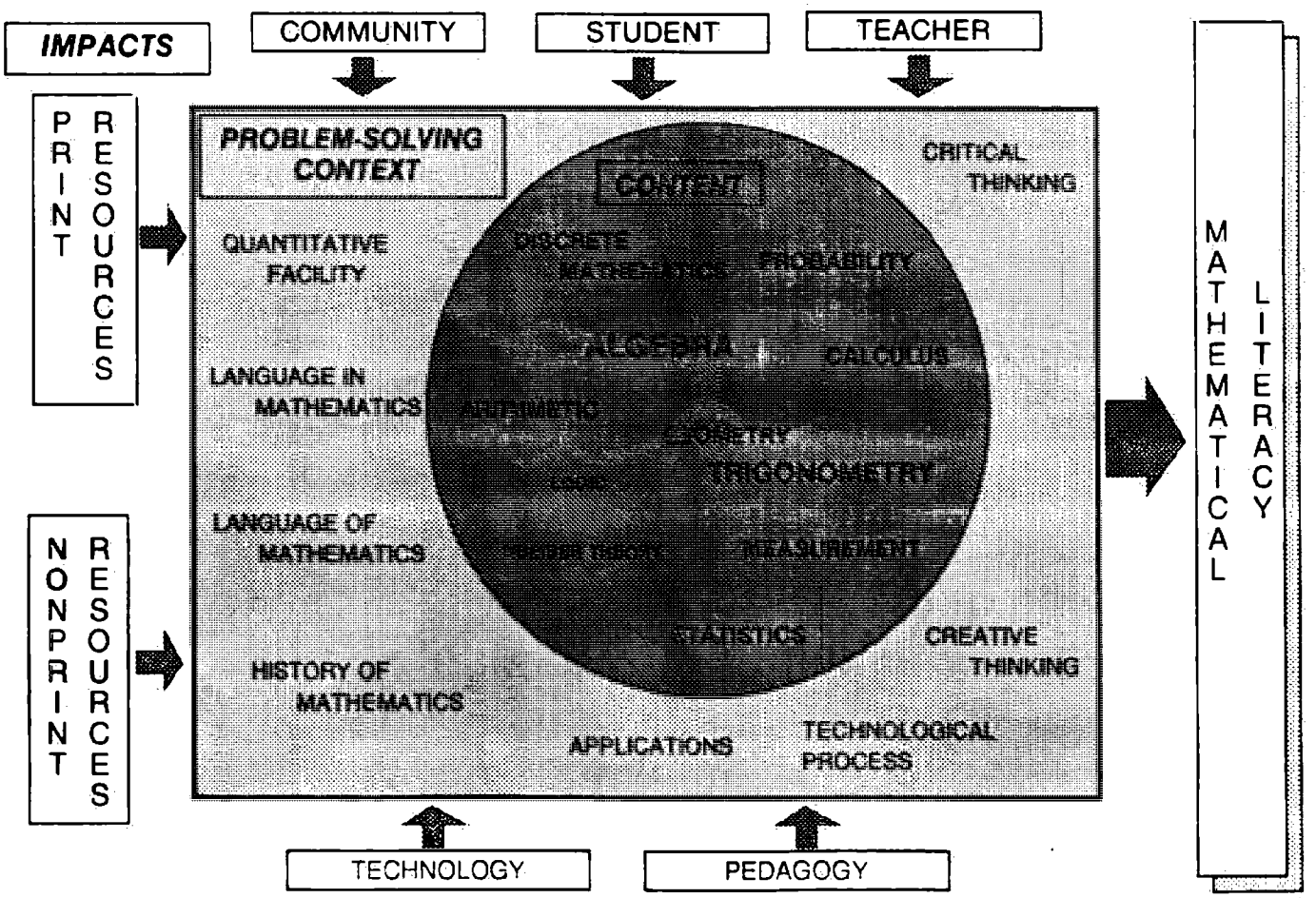
The Senior High School Mathematics Program, by providing course sequences commensurate with differing abilities, interests and aspirations, is designed to enable students to have success in mathematics. As well, it reflects the changing needs of society, and provides students with the mathematical concepts, skills and attitudes necessary to cope with the challenges of the future.

B. GENERAL LEARNER EXPECTATIONS

Upon conclusion of their senior high mathematics education, students will be expected to be mathematically literate. More specifically, *students will be expected to*

- have achieved understanding of the basic mathematical concepts, and developed the skills and attitudes needed to become responsible and contributing members of society
- apply basic mathematical skills and concepts in practical situations
- have developed the skills, concepts and attitudes that will ensure success in the mathematical situations that occur in future educational endeavours, employment and everyday life
- have developed the skills, concepts and attitudes that will enable the acquisition of mathematical knowledge beyond the conclusion of secondary education
- have developed critical and creative thinking skills
- be able to communicate mathematical ideas effectively
- understand how mathematics can be used to investigate, interpret and make decisions in human affairs
- understand how mathematics can be used in the analysis of natural phenomena
- understand and appreciate the positive contributions of mathematics, as a science and as an art, to civilization and culture.

A Model for Mathematical Literacy



A Model for **Mathematical Literacy** outlines the factors that effect what and how students learn as they become mathematically literate. The model is fluid in that the content can be learned within a problem-solving context which engages any of a number of vehicles as the learning focus. At the same time, the students involved in the learning situation are affected in what and how they learn by forces which have impact upon them.

The **Content** of the high school mathematics program is that body of knowledge that is to be acquired by students. In the various high school mathematics courses, it is made up of topics that can be categorized into one or more of the listed strands.

Within each course in the high school mathematics program, students will focus on problem solving. The **Problem-Solving Context** refers to the instructional emphases within which the specific content expectations can be acquired. The various entries indicated within the model suggest processes that belong to the problem-solving context and may be used by students as vehicles for learning the content.

The **Impacts** on the Problem-Solving Context are those skills, attitudes and experiences that are possessed by the students and teachers involved, as well as the resources they may use throughout the learning process. They include the influence exerted by the culture and beliefs of the community as reflected by the school. The effectiveness of the context in enabling a student to acquire the content is dependent upon the skilful management by the teacher of those items that impact upon a student's learning.

Mathematical literacy refers to the ability to use mathematical skills, concepts and processes to address successfully the mathematical situations which one encounters.

C. SPECIFIC LEARNER EXPECTATIONS

The specific learner expectations are what a student is expected to accomplish in a particular course. For mathematics these include attitudes, concepts and skills, as well as expectations about problem solving and the use of technology.

Attitudes are general dispositions toward mathematics education and social issues. Attitude expectations are listed at the beginning of each course and stated as "Students will be encouraged to develop...".

A concept is an abstract or general idea about specific instances that have common properties or an identifiable relationship to one another. Each unit is broken into a number of concepts which are the major ideas to be addressed. Concepts may be presented as mathematical definitions or theorems, or as statements of mathematical ideas or abstractions. In the course of study, concepts are numbered with a single digit and stated as "Students will be expected to demonstrate an understanding that...".

Supporting each concept are a number of skills. Skills are intellectual or physical capabilities that will be developed within the context of the particular concept. In the program of studies, skills are numbered with two digits separated by a period, (n.m) and stated as "Students will be expected to...".

In addition, related to particular concepts and skills, are problem-solving expectations. These expectations are of two general types. Some identify specific types of problems which must be solved while others identify problem-solving behaviours or techniques to be learned by students. Problem-solving expectations occur throughout the specific learner expectations. They suggest that students should have the background skills and knowledge necessary to achieve the expectations successfully, using problem-solving techniques. In the program of studies, problem-solving expectations are numbered with three digits separated by periods (n.m.l).

Technology expectations identify specific areas in which calculator and/or computer technology will be applied by students as tools to be used for calculation, manipulation or graphing, or to aid in the analysis of problems. In the program of studies, technology expectations are numbered with three digits separated by periods (n.m.l).

Throughout the learner expectations, the words verify and prove appear. For the purposes of the Senior High Mathematics Program, they are interpreted as:

- Verify - to substantiate the validity of an operation, solution, formula or theorem through the use of examples which may or may not be generalized;
- Prove - to substantiate the validity of an operation, solution, formula or theorem in general and to provide logical arguments for each step in the process.

The major part of the content of each high school mathematics course consists of topics required of all students who take the course. The required content comprises 80% of the course and contains the concepts, skills and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations in regard to problem solving and the use of technology.

Each course will include a compulsory component comprising 20% of the course, made up of elective material which will be consistent with the content and expectations of the required component. The purpose of the elective is to provide for enrichment, remediation, or innovative or experimental presentations or activities. It is not intended to provide acceleration or advanced placement. However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

Through the use of problems and applications, students will be able to see the need for certain concepts and mathematical procedures, to understand concepts and acquire facility with procedures, and to apply and review concepts and procedures already learned. The emphasis on problem solving in high school mathematics has shifted from finding answers to routine "word problems", to the application of mathematical concepts, skills and processes to develop and discover mathematics within a wide variety of problem situations.

Problem solving has been integrated into all of the courses in the Senior High Mathematics Program and is explicitly mentioned throughout the learner expectations. In addition, specific expectations for problem solving are included at the beginning of each course. It is not intended that problem solving be presented as units of study; however, it is expected that upon completion of the course, students will have acquired all of the listed skills and concepts .

Students will be expected to learn how and when to use a calculator and have a demonstrated proficiency in estimation and mental arithmetic. To use calculators effectively, they must be able to judge the reasonableness of an answer and understand the importance of making a judgment about the result of a calculation.

Throughout the learner expectations, specific applications of calculators and computers are identified. These define explicitly the areas in which students are expected to use technology in their study of mathematics. Many of these expectations indicate that a particular technology be used in an investigation or for analysis. It is in these situations that the use of technology enables students to engage in critical and creative thinking and problem solving.

Evaluation of students in the Senior High Mathematics Program will involve assessment of the level of achievement of all of the learner expectations, including concepts, skills and attitudes, as well as problem solving and technology expectations. For more information regarding evaluation consult the Teacher Resource Manual for Senior High Mathematics.

Attitudes

Within the study of the content of this course, it is expected that *students will be encouraged to*

- be confident in their mathematical knowledge and in their ability to acquire new knowledge;
- demonstrate persistence, resolve, flexibility and ingenuity in finding the solution to problems;
- develop an attitude of intellectual curiosity and openness to new ideas, insights and change in the pursuit of mathematical knowledge;
- exhibit an attitude of curiosity and spontaneity and appreciate creativity and innovation in representing situations mathematically;
- value and be aware of the importance of communication skills in mathematics;
- appreciate and develop a positive attitude toward the usefulness of computational competence, mathematical processes and problem solving skills which are used in the decision making and modeling processes in our society;
- value the contributions of mathematics to our culture and civilization.

Problem Solving

Problem solving is to be integrated into all areas of the high school mathematics program because a major purpose of studying mathematics is to learn to solve problems. A student who has successfully completed this course will understand that thinking and problem-solving skills are required in order to explain the world, and that mathematics can provide some of the tools for problem solution.

Students will understand that problems can be solved in many ways, and that problems can be routine, requiring only the application of a known procedure or algoarithm, or non-routine, requiring the development of a process or the conducting of an investigation to solve them.

Students will be expected to use a variety of procedures to help them understand mathematical problems. In particular, they will be expected to

- read the problem thoroughly
- identify and clarify key components
- restate the problem using familiar terms
- evaluate the given information as to whether it is insufficient or extraneous
- interpret pictures, charts and graphs
- determine any hidden assumptions
- ask relevant questions
- identify given, needed and wanted information
- diagram or model the problem situation
- use suitable notation
- determine valid inferences.

Students will be expected to develop a variety of strategies for use in the solution of mathematical problems. In particular, they will be expected to

- conduct an investigation
- use estimation and approximation
- develop equations or use formulae
- use flow charts
- make lists and charts
- look for patterns
- work backward
- break the problem into smaller parts
- look for a simpler or related problem
- make diagrams or models
- use manipulatives
- choose and sequence a series of mathematical operations
- sketch the graph of a problem situation
- establish procedures to gather and organize data.

Students will be expected to develop a variety of skills which can be used to carry out the plan for the solution of a problem. In particular, they will be expected to

- apply selected strategies.
- present ideas clearly
- document the solution process
- utilize appropriate group behaviours
- use calculators and computers
- evaluate problem-solving strategies for effectiveness
- alter or abandon non-productive strategies
- search for additional information
- ask questions
- be open to inspirations, intuitions and "bright ideas".

Students will be expected to employ a variety of skills to help them look back over the solution of a problem. In particular, they will be expected to

- determine the reasonableness of an answer
- explain the solution in oral or written form
- consider the possibility of additional solutions
- search for other strategies and processes of solution
- create and solve similar problems
- note the characteristics that will be identifiable in similar problems
- make a generalization.

Numeration

Numeration concepts and calculation skills are basic in order to successfully address any mathematical situation. Since it is the intent to integrate the development of these concepts and skills into the other expectations of this course, it is not expected that numeration will be taught as a topic on its own.

1. *Students will be expected to* demonstrate an understanding that computation requires skills in addition, subtraction, multiplication and division with whole numbers, integers and rational numbers and can be done mentally, with paper and pencil, and with electronic calculators, as appropriate to a particular situation.

   Mental Computation

   Students will be expected to

   1.1 add and subtract whole numbers, integers and rational numbers (limit: add three single-digit integers; subtract two single-digit integers; add and subtract two multiples of 5 and 10 up to 100; add and subtract fractions with common denominators; add and subtract tenths and hundredths written in decimal form)
   1.2 multiply whole numbers, integers and rational numbers (limit: single digit numbers, multiples of 5 and 10 multiplied by a single-digit multiplier; double any number up to 50; multiply whole numbers, integers and decimals by powers of 10)
   1.3 square integers up to 10 and calculate square roots of integral perfect squares up to 100
   1.4 divide whole numbers, integers and rational numbers (limit: divide integers to 100 which are perfect multiples of single-digit divisors; halve even integers to 50; divide whole numbers, integers and decimals by powers of 10)
   1.5 determine whether a whole number or integer is divisible by 2, 3, 4, 5 or 10
   1.6 transform fractions to percents (limit: denominators of 2, 3, 4, 5, 10, 100) and whole number percents
   1.7 calculate the percent of given whole number quantities (limit: 1%, 10%, 25%, 50%, 100%, 200%)
   1.8 recognize "families" of fractions (tenths, twelfths, sixteenths) and transform fractions within families to common denominators.

   Paper and Pencil Computation

   Students will be expected to

   1.9 add and subtract whole numbers, integers and rational numbers (limit: add three 2-digit integers; subtract 3-digit integers; add and subtract two fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 16; add three 2-digit decimals; subtract 2-digit decimals)

1.10 multiply whole numbers, integers and rational numbers (limit: 2-digit integers, 2-digit decimals, fractions with single-digit denominators and denominators which are multiples of 10)
1.11 divide whole numbers, integers and rational numbers (limit: 3-digit integers which are perfect multiples of single-digit divisors; halve even numbers up to 500)
1.12 calculate the percent of given quantities (limit: 5%, 15%, 25% and multiples of 10%)
1.13 calculate the value of a power (limit: whole number base <10 and exponent of 2 or 3)
1.14 transform mixed numbers to decimals and percents (limit: denominators of 2, 3, 4, 5, 6, 8, 10).

Calculation with Calculators

Students will be expected to

1.15 calculate the percent of given quantities
1.16 calculate the value of a power (limit: integral base and whole number exponent)
1.17 calculate the square root of any positive rational number
1.18 transform any rational number to decimal and percent form.

2. *Students will be expected to* demonstrate an understanding that rounding, estimation and approximation are fundamental to facility in calculation.

Students will be expected to

2.1 round given values to a specified level of precision
2.2 round solutions to an appropriate level of precision
2.3 use estimation techniques to predict solutions to calculations.

3. *Students will be expected to* demonstrate an understanding that checking an answer for reasonableness and accuracy is a fundamental step in the calculation process and that the level of accuracy required is dependent upon the particular situation.

Students will be expected to

3.1 use estimation techniques to check a calculation for reasonableness
3.2 determine if the accuracy of a calculated answer is adequate for a given situation.

Ratio and Proportion

1. *Students will be expected to* demonstrate an understanding that a ratio is a way of comparing the relative sizes of two quantities.

Students will be expected to

1.1 compare two quantities using ratios in the form $\frac{a}{b}$, a:b, and a to b.

2. *Students will be expected to* demonstrate an understanding that a rate is a ratio between quantities which have different units.

Students will be expected to

2.1 calculate rates and unit rates

2.1.1 solve problems involving the use of rates and unit rates.

3. *Students will be expected to* demonstrate an understanding that proportions are equations made up of equivalent ratios.

   Students will be expected to

   3.1 calculate the unknown value in a proportion

      3.1.1 solve problems using proportions.

4. *Students will be expected to* demonstrate an understanding that a percent is a ratio in which a quantity is compared to 100.

   Students will be expected to

   4.1 express ratios as percents and decimals and vice-versa
   4.2 solve for the unknown element in a percentage calculation

      4.2.1 solve problems which involve the use of percents.

Statistics and Probability

1. *Students will be expected to* demonstrate an understanding that data collected from a sample must be organized, presented and analyzed in order that valid inferences can be drawn.

   Students will be expected to

   1.1 use the terms sample and population
   1.2 determine a suitable sample from a given population

      1.2.1 assess the suitability of a sample chosen from a given population.

   1.3 organize data using tally sheets, frequency tables and stem-and-leaf plots

      1.3.1 design and carry out a simple statistical project involving the collection, organization, presentation and analysis of data gathered from an appropriate sample

   1.4 interpret and analyze data represented by pictographs, circle graphs, bar graphs, line graphs, and histograms
   1.5 identify the characteristics of a misleading graphical display
   1.6 determine and calculate appropriate measures of central tendency

      1.6.1 solve problems which involve the use of an appropriate measure of central tendency in the analysis of sets of data

   1.7 make inferences and generalizations about a population from which a sample was taken.

2. *Students will be expected to* demonstrate an understanding that a probability is a numerical measure of the likelihood of an event and can take values from 0 through 1.

   Students will be expected to

   2.1 list the sample spaces for simple experiments or simulations
   2.2 use counting techniques such as tree diagrams to find the number of possible outcomes of an experiment

2.3 express the probabilityof an event from a given situation, simple experiment or simulation

2.3.1 perform an investigation to find the experimental probability of an event.

Geometry

1. *Students will be expected to* demonstrate an understanding that two- and three-dimensional geometric figures can be classified and have characteristics which uniquely define them.

Students will be expected to

1.1 classify polygons according to the number of sides
1.2 classify triangles and quadrilaterals by sides and angles
1.3 determine the sum of the interior angles of a polygon

1.3.1 perform an investigation to determine the sum of the interior angles of any polygon

1.4 classify the following common geometric solids: rectangular prisms, pyramids, cylinders, cones and spheres

1.4.1 perform an investigation to determine the relationship amongst the edges, faces and vertices of polyhedra
1.4.2 draw nets which define simple polyhedra
1.4.3 use nets to construct simple polyhedra

1.5 identify and define the following terms associated with a circle such as the following: radius, diameter, circumference, semicircle.

2. *Students will be expected to* demonstrate an understanding that geometric figures can be defined uniquely and that these unique characteristics can be reproduced using construction techniques and geometric instruments.

Students will be expected to

2.1 construct a line segment of given length or congruent to a given segment
2.2 draw an angle of a given size or construct an angle congruent to a given angle
2.3 construct the perpendicular bisectors of segments and angles
2.4 draw regular polygons
2.5 construct a circle of given radius
2.6 use a combination of construction techniques

2.6.1 construct angles of 90°, 45°, 60° and 30°
2.6.2 use construction techniques in the construction or reproduction of geometric figures or designs.

3. *Students will be expected to* demonstrate an understanding that similar geometric figures have the same shape but may differ in size, that corresponding angles are equal and that the lengths of corresponding sides are proportional.

Students will be expected to

3.1 use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides

3.1.1 solve problems using the properties of similar polygons

3.2 recognize congruency as similarity in which the size of the figures is the same.

4. *Students will be expected to* demonstrate an understanding of the Pythagorean theorem.

Students will be expected to

4.1 use the Pythagorean theorem to find the length of a side in a right triangle

4.1.1 solve problems involving right triangles by using the Pythagorean theorem.

5. *Students will be expected to* demonstrate an understanding that angles formed by intersecting lines relate to each other.

Students will be expected to

5.1 identify parallel, intersecting and perpendicular lines
5.2 identify and recognize the characteristics of the relationship between supplementary, complementary and vertical angles

5.2.1 perform an investigation to determine the properties of the angles between intersecting lines
5.2.2 solve problems involving the relationships amongst intersecting and parallel lines and the angles they form to solve problems involving the measure of unknown angles.

Measurement

1. *Students will be expected to* demonstrate an understanding that standard units of measure for length, mass, and volume provide consistency and accuracy and that the most widely used and accepted measurement system is the SI (metric) system.

Students will be expected to

1.1 recognize the basic SI units of measurement for length (metre (m)), mass (kilogram (kg), (tonne (t)), and volume of liquids or gases (litre (L))
1.2 use the appropriate SI measures for length, mass and liquid/gas volume. (limit: kg, g, L mL, m, cm, mm, km)
1.3 express equivalences between units of length, of mass and of volume
1.4 recognize non-metric units of measurement still in common use.

2. *Students will be expected to* demonstrate an understanding that perimeter is the measure of the distance around a geometric figure.

Students will be expected to

2.1 calculate the perimeter of polygons

2.1.1 calculate the perimeter of irregular shapes and objects

2.2 calculate the circumference of a circle by using a formula

2.2.1 solve problems which involve finding the circumference of circles and circular regions.

3. *Students will be expected to* demonstrate an understanding that area is the number of unit squares which are enclosed within the boundaries of a geometric figure.

   Students will be expected to

   3.1 calculate the area of triangles and quadrilaterals by using formulas (limit: triangles, quadrilaterals)

      3.1.1 solve problems which involve finding the area of triangles and rectangles

   3.2 find the area of a circle by using a formula

      3.2.1 solve problems which involve finding the area of circles.

4. *Students will be expected to* demonstrate an understanding that volume is the number of unit cubes which are within a geometric solid.

   Students will be expected to

   4.1 calculate the volume of rectangular prisms and cylinders by using formulas

      4.1.1 solve problems which involve calculating the volume of rectangular prisms and cylinders.

5. *Students will be expected to* demonstrate an understanding that time can be measured in seconds, minutes, hours, days, weeks, months and years.

   Students will be expected to

   5.1 change units of time from one form to another
   5.2 tell time using a 24-hour clock
   5.3 write times using proper 24-hour notation
   5.4 perform calculations using units of time.

6. *Students will be expected to* demonstrate an understanding that temperature can be measured in degrees.

   Students will be expected to

   6.1 measure temperatures using the Celsius scale
   6.2 perform calculations involving the use of temperature measured in Celsius.

Algebra and Graphing

1. *Students will be expected to* demonstrate an understanding that algebraic expressions can be used to represent situations where one or more values can vary.

   Students will be expected to

   1.1 write first degree algebraic expressions to illustrate simple mathematical situations with a maximum of 2 variables.

1.2 evaluate algebraic expressions for replacement values of the variables

1.2.1 solve problems which involve the evaluation of expressions for values of the variables

1.3 solve first degree equations in one variable through the use of guess-and-test techniques, concrete manipulatives and formal methods (limit: $x+a=b$, $ax=b$, $ax+b=c$, $\frac{x}{a}=b$)

1.3.1 solve problems which can be represented by first degree equations in one variable
1.3.2 use calculators to aid in the solution of equations.

2. *Students will be expected to* demonstrate an understanding that two variables can be related and that those relationships can be expressed by sets of ordered pairs, equations and graphs.

Students will be expected to

2.1 describe verbally and in writing the relationships between two variables

2.1.1 determine the algebraic relationship between two variables by informal means such as guess-and-test

2.2 plot points on a coordinate plane
2.3 plot the points and sketch the graphs which describe the relationship between two variables
2.4 construct tables of values which satisfy given linear equations in two variables
2.5 sketch graphs from tables of values which satisfy a given linear equation in two variables
2.6 read and interpret the graphs which describe the relationship between two variables

2.6.1 analyze graphs which describe the relationship between physical variables by the use of interpolation and extrapolation techniques.

D. BASIC LEARNING RESOURCES

Price, Jack, Olene Brame, Michael Charles and Miriam Clifford. Applications of Mathematics. Weston, Ontario: Merril Publishing Company, 1989.